دورة العام ٢٠١٦ الاستثنائية	امتحانات الشهادة المتوسطة	وزارة التربية والتعليم العالي
الجمعة ٢٩ تموز ٢٠١٦		المديرية العامة للتربية
		دائرة الامتحانات الرسمية

	الاسم:	مسابقة في مادة الرياضيات	عدد المسائل: خمسة
	الرقم:	المدة: ساعتان	

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.

- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I - (2 points)

Consider the three numbers A, B and C:

$$A = \frac{1}{3} + \frac{7}{6} \div \frac{5}{3} \; ; \; B = \frac{5 \times 10^{-2} \times 7 \times 10^5}{2 \times 10^7} \text{ and } C = \sqrt{45} - 4\sqrt{5} + 3\sqrt{125} .$$

In what follows, show all details of calculation:

1) Calculate A and write the answer as a fraction in the simplest form.

2) Calculate B and write the answer in scientific notation.

3) Write C in the form $a\sqrt{5}$ where a is a natural number.

II - (3 points)

The following two questions 1) and 2) are independent.

1) Solve the following inequality and represent the solution on a number line with origin O:

$$4x - \frac{3}{2} \le \frac{5}{2}x + 3$$

2) A box contains 400 balls, distributed as follows:

- 30% of these balls are red
- 108 balls are green
- The remaining balls are white.

a. Find the percentage of green balls.

b. Calculate the number of white balls.

III - (3 points)

Given: $E(x) = 5(x − 1)(x + 2) − (x + 2)^2 + 3(x + 5)$.

1) Show that $E(x) = 4x^2 + 4x + 1$.

2) Solve the equation $E(x) = 1$.

3) Consider $H(x) = 9x^2 - (2x + 1)^2$.

a. Show that $H(x) = (5x + 1)(x − 1)$.

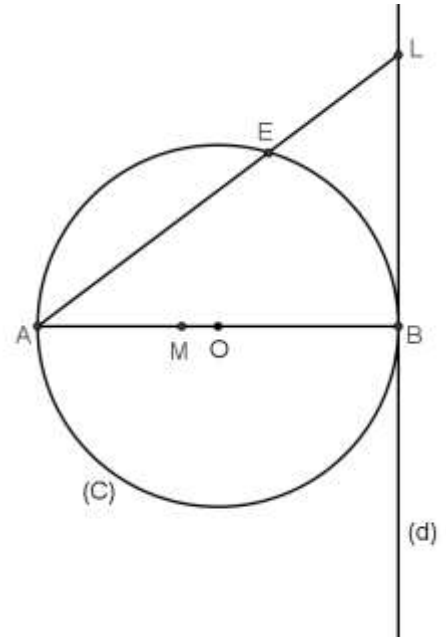**b.** Solve the equation $H(x) = 0$.

1

IV- (6 points)

In an orthonormal system of axes (x'Ox , y'Oy), consider the line (d) with equation $y = -2x + 3$ and the points $A(0; -2)$, $E(6; 1)$ and $G(0; 3)$.

1) Plot the points A, E and G.

2) Verify that G is a point on line (d), then draw (d).

3) a. Show that $y = \dfrac{1}{2}x - 2$ is the equation of line (AE).

    b. Prove that the lines (d) and (AE) are perpendicular.

    c. Verify, by calculation, that B $(2; -1)$ is the intersection point of lines (d) and (AE).

    d. Prove that GBE is a right isosceles triangle.

4) Denote by M the translate of E under the translation with vector $\overrightarrow{BG}$.

    a. Prove that the quadrilateral BEMG is a square.
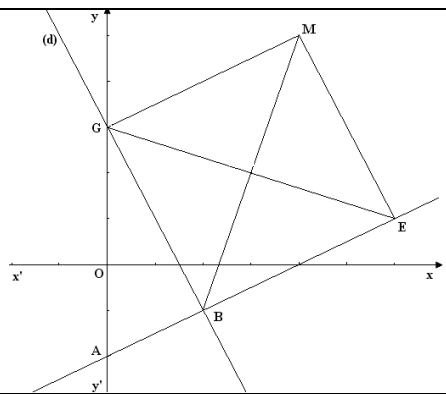
    b. Calculate BM.

V- (6 points)

In the adjacent figure:

- (C) is a circle with center O and radius 5 cm
- [AB] is a diameter of this circle
- (d) is the tangent at B to (C)
- L is a point on (d) such that BL = 7.5 cm
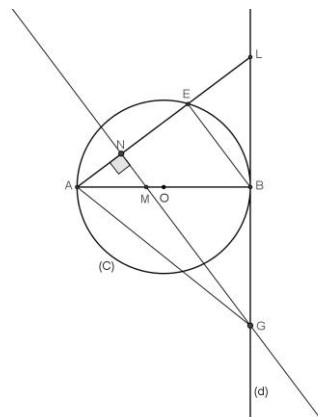- M is a point on [AB] such that AM = 4cm.

1) Copy the figure.

2) Calculate the length AL.

3) Calculate $\cos BAL$.

4) The line (AL) intersects the circle (C) at E.

    a. Prove that the two triangles ABL and BEL are similar.

    Write the ratio of similarity.

    b. Deduce that EL = 4.5 cm.

5) The perpendicular through M to (AL) intersects [AL] at N and (d) at G.

    a. Use $\cos BAL$ in triangle MAN to verify that AN = 3.2 cm.

    b. Prove that: $\dfrac{EB}{NG} = \dfrac{15}{31}$.

Question I		
Part	**Correction**	**Note**
1	$A = \dfrac{1}{3} + \dfrac{7}{6} \div \dfrac{5}{3} = \dfrac{1}{3} + \dfrac{7}{6} \times \dfrac{3}{5} = \dfrac{1}{3} + \dfrac{7}{10} = \dfrac{31}{30}$. **0.25 +0.25**	**0.50**
2	$B = = \dfrac{5 \times 10^{-2} \times 7 \times 10^{5}}{2 \times 10^{7}} = \dfrac{35 \times 10^{3}}{2 \times 10^{7}} = \dfrac{35 \times 10^{-4}}{2} = 17.5 \times 10^{-4} = 1.75 \times 10^{-3}$ **0.25 +0.25+0.25**	**0.75**
3	$C = \sqrt{45} - 4\sqrt{5} + 3\sqrt{125} = 3\sqrt{5} - 4\sqrt{5} + 15\sqrt{5} = 14\sqrt{5}$. **0.5 +0.25**	**0.75**
Question II		
1	$4x - \dfrac{3}{2} \le \dfrac{5}{2}x + 3$; $\dfrac{8x}{2} - \dfrac{3}{2} \le \dfrac{5x}{2} + \dfrac{6}{2}$; $8x - 3 \le 5x + 6$; $8x - 5x \le 3 + 6$; $3x \le 9$; $x \le 3$. **0.25 +0.25+0.25 +0.25**	**1** **0.5**
2.a	The percentage of green balls is : $\dfrac{108}{400} \times 100 = 27\%$	**0.5**
2.b	The percentage of white balls is: $100 - (27 + 30) = 100 - 57 = 43\%$	**0.5**
	The number of white balls is: $\dfrac{43 \times 400}{100} = 172$	**0.5**
Question III		
1	$E(x) = 5(x^2 + x - 2) - (x^2 + 4x + 4) + 3x + 15 = 4x^2 + 4x + 1$. **- 0.25** (MISTAKE)	**1**
2	$E(x) = 1$ so $4x^2 + 4x = 0$. where $4x(x + 1) = 0$; $x = 0$ or $x = -1$. **0.25 +0.25**	**0.5**
3.a	$H(x) = 9x^2 - (2x + 1)^2 = (3x + 2x + 1)(3x - 2x - 1) = (x - 1)(5x + 1)$.). **0.5 +0.5**	**1**
3.b	$H(x) = 0$; $(x - 1)(5x + 1).= 0$. So $x = 1$ or $x = -\dfrac{1}{5}$	**0.5**
Question IV		
1		**0.5**
2	$y_G = -2x_G + 3$ then $3 = -2(0) + 3$ $3 = 3$ so G is a point on (d) Two points are enough to draw a line : <table><tr><td>x</td><td>0</td><td>1</td></tr><tr><td>y</td><td>3</td><td>1</td></tr></table>	**0.5** **0.25**
3.a	$y_E = \dfrac{1}{2}x_E - 2$ $1 = \dfrac{1}{2}(6) - 2$ $y_A = \dfrac{1}{2}x_A - 2$ $-2 = \dfrac{1}{2}(0) - 2$ $1 = 1$ so E is a point on (AE). $-2 = -2$ so A is a point on (AE).	**0.5** **0.5**
3.b	$a_{(d)} \times a_{(AE)} = -2 \times \dfrac{1}{2} = -1$ So (d) $\perp$ (AE).	**0.5**

3.c	$y_B = \dfrac{1}{2} x_B - 2$ $-1 = \dfrac{1}{2}(2) - 2$ $-1 = -1$ so B is a point on (AE).	$y_B = -2x_B + 3$ $-1 = -2(2) + 3$ $-1 = -1$ so B is a point on (d).	**0.5** **0.25**
3.d	$GBE = 90^o$ (since (d) $\perp$ (AE)) $BG = 2\sqrt{5}$. $BE = 2\sqrt{5}$. So GBE is a right isosceles triangle of vertex B.		**0.25** **0.25** **0.25** **0.25**
4.a	We have: $\overrightarrow{BG} = \overrightarrow{EM}$ so BEMG is a parallelogram And since : $GBE = 90^o$ then it is a rectangle and: $BG = BE$ so it is a rhombus.therefore: BEMG is a square.		**0.25** **0.25** **0.25**
4.b	$BM = GE = \sqrt{(6-0)^2 + (1-3)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$ u (diagonals in a square are equal)		**0.5**
Question V			
1			**0.5**
2	Using pythagorean : $AL^2 = AB^2 + BL^2 = 100 + 7.5^2 = \dfrac{625}{4}$ so: $AL = 12.5$ cm		**0.5** **0.25**
3	$\cos BAL = \dfrac{AB}{AL} = \dfrac{10}{12.5} = \dfrac{4}{5}$ **0.5+0.25**		**0.5**
4.a	The two triangles ABL and LBE have: • L (common angle) • $B = E = 90^0$ (ABE is an inscribed triangle in a semi-circle) So they are similar. $S_{BEL}^{ABL}; \dfrac{AB}{BE} = \dfrac{BL}{EL} = \dfrac{AL}{BL} = \dfrac{12.5}{7.5} = \dfrac{5}{3}$		**0.5** **0.5** **0.5**
4.b	Consider $\dfrac{BL}{EL} = \dfrac{5}{3}$; $\dfrac{7.5}{EL} = \dfrac{5}{3}$; $EL = 4.5$ cm		**0.25** **0.25**
5.a	$\cos BAL = \dfrac{4}{5}$ and $\cos BAL = \cos NAM = \dfrac{AN}{AM} = \dfrac{4}{5}$ so $AN = 3.2$ cm		**0.25** **0.25** **0.5**
5.b	$(EB)//(NG)$ Using Thales' $\dfrac{LE}{LN} = \dfrac{EB}{NG} = \dfrac{4.5}{9.3} = \dfrac{15}{31}$ (LN $= 12.5 - 3.2 = 9.3$ cm)		**0.25** **0.5** **0.25**