

الاسم:
الرقم:مسابقة في مادة الرياضيات
المدة: ساعتان

عدد المسائل : خمسة

إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يجب طبع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I. (3 points)

Consider five distinct points O, I, K, L and S such that:

$$OI = \frac{5}{2} - \cos 60^\circ, \quad OK = \frac{2 \times 10^3 \left[(2 \times 10^{-1})^2 + (6 \times 10^{-2}) \right]}{5 \times 10^{-1} \times 2 \times 10^2}$$

$$OL = 20 \left(\frac{5}{6} - \frac{2}{15} - \frac{3}{5} \right) \quad \text{and} \quad OS = \frac{(\sqrt{10} - \sqrt{2})(\sqrt{10} + \sqrt{2})}{\sqrt{16}}.$$

- 1) Calculate, in showing the details of calculation OI, OK, OL and OS and write each result in the form of a natural number.
- 2) Show that the points I, K, L and S belong to the same circle with center O.

II. (3 points)

The following table represents the grades over 60 of 20 students:

Grade	45	48	52	56	58	60
Frequency	3	6	4	2	1	4
Increasing cumulative frequency						

- 1) What is the relative frequency of the grade 52?
- 2) Complete the previous table.
- 3) Plot the increasing cumulative frequency polygon.
- 4) What is the percentage of students having a grade greater than 57 ?
- 5) Determine the average grade (mean) of the students.

III. (4 points)

Given $E(x) = (2x - 1)^2 + (x - 2)(1 - 2x)$ and $F(x) = ax^2 + bx - 2$.

- 1) Factorize $E(x)$.
- 2) Calculate a and b such that $F(1) = 5$ and $F(-2) = 20$.
- 3) Let $Q(x) = 6x^2 + x - 2$. Verify that $Q(x) = (2x-1)(3x+2)$.
- 4) Let $P(x) = \frac{E(x)}{Q(x)}$.
 - a) Determine the values of x so that P(x) is defined. Then simplify P(x).
 - b) Solve the equation $P(x) = 0$.
 - c) Does the equation $P(x) = \frac{3}{7}$ have a solution? Justify.

IV. (5 points)

In an orthonormal system $x'Ox, y'Oy$ with 1cm as unit of length, consider the points $A(-2; 3), B(1; -1), C(9; 5)$, and the line (d) with equation $y = 2x - 13$.

- 1) Locate the points A, B, C and plot the line (d) .
- 2) Calculate the coordinates of N the intersection point of (d) with the axis $x'Ox$.
- 3) Prove that the triangle ABC is right angled at B .
- 4) Let M be the midpoint of $[AC]$, calculate the coordinates of M .
- 5) Show that N is the translate of C by the translation with vector \overline{MB} .
- 6) Prove that the quadrilateral $BMCN$ is a rhombus.

V. (5 points)

In the opposite figure:

- (S) is a semicircle with center O and radius R
- $[EF]$ is the diameter of (S)
- A is a point on (EF) so that $OA = 2R$
- (d) is a variable line through A that intersects (S) at B and C
- The tangents at B and C to (S) intersect at M .

1) Justify that (OM) is the perpendicular bisector of $[BC]$.

2) (OM) intersects $[BC]$ at I . let P be the orthogonal projection of M on (OA) .

a. Show that the triangles OIA and OMP are similar.

b. Prove the relation $OA \times OP = OM \times OI$.

3) a. Let (S') be the circle circumscribed about the triangle CIM . Show that (OC) is tangent to (S') .

b. Use two similar triangles to prove that $OM \times OI = R^2$.

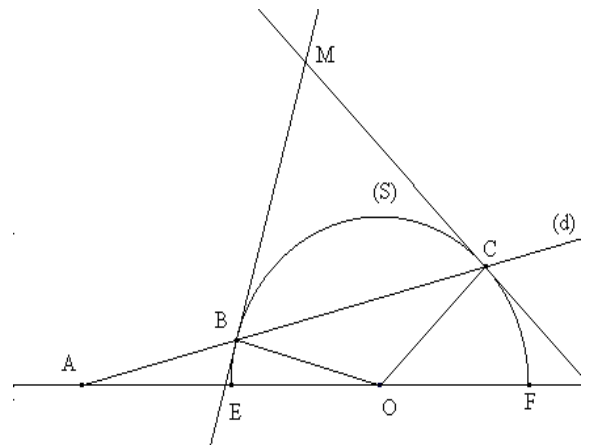
c. Calculate OP in terms of R . Deduce the locus of M as (d) varies.

4) **In this question, suppose that $OBMC$ is a square.**

a. Calculate the lengths BC and MP in terms of R .

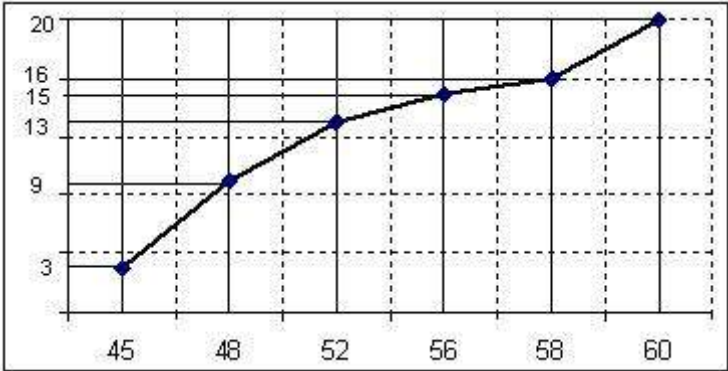
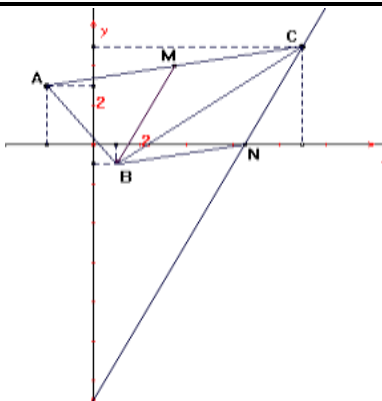
b. Calculate the exact value of $\tan \widehat{MAP}$.

c. Calculate the value of \widehat{MAP} to the nearest degree.



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مشروع معيار التصحيح

Q.	Corrigé	Note																					
I	$OI = \frac{5}{2} - \frac{1}{2} = 2.$ $OL = 20 \left(\frac{25 - 4 - 18}{30} \right) = 20 \left(\frac{3}{30} \right) = 2.$ $OS = \frac{(\sqrt{10} - \sqrt{2})(\sqrt{10} + \sqrt{2})}{\sqrt{16}} = \frac{10 - 2}{4} = 2.$ $OK = \frac{2 \times 10^3 [(4 \times 10^{-2}) + (6 \times 10^{-2})]}{5 \times 2 \times 10} =$ $\frac{2 \times 10^3 [10^{-2}(4 + 6)]}{10^2} = \frac{2 \times 10 \times (10^{-2} \times 10)}{1} = \frac{2 \times 10 \times 10^{-1}}{1} = 2.$	2.50																					
	<p>2 $OI = OK = OL = OS = 2$, therefore the points I, K, L and S belong to the same circle with center O and radius 2.</p>	0.50																					
II	<p>1 Relative frequency $52 = \frac{4}{20} = 0.2$</p>	0.50																					
	<table border="1"> <tr> <td>Grade</td> <td>45</td> <td>48</td> <td>52</td> <td>56</td> <td>58</td> <td>60</td> </tr> <tr> <td>Frequency</td> <td>3</td> <td>6</td> <td>4</td> <td>2</td> <td>1</td> <td>4</td> </tr> <tr> <td>Commulative frequency</td> <td>3</td> <td>9</td> <td>13</td> <td>15</td> <td>16</td> <td>20</td> </tr> </table>	Grade	45	48	52	56	58	60	Frequency	3	6	4	2	1	4	Commulative frequency	3	9	13	15	16	20	0.50
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	Frequency	3	6	4	2	1	4																
	Commulative frequency	3	9	13	15	16	20																
<p>3</p> 	0.75																						
<p>4 $\frac{5}{20} \times 100 = 25\%$</p>	0.75																						
<p>5 $\bar{x} = 52.05.$</p>	0.50																						
III	<p>1 $E(x) = (2x-1)(2x-1-x+2) = (2x-1)(x+1)$</p>	0.50																					
	<p>2 $F(1) = a + b - 2 = 5; a + b = 7$ $F(-2) = 4a - 2b - 2 = 20$; then $a = 6$ and $b = 1$</p>	1.25																					
	<p>3 $(2x-1)(3x+2) = 6x^2 + x - 2.$</p>	0.50																					
	<p>4a a) $P(x) = \frac{E(x)}{Q(x)} = \frac{(2x-1)(x+1)}{(2x-1)(3x+2)}$ is defined if $x \neq \frac{1}{2}$ and $x \neq -\frac{2}{3}$; then $P(x) = \frac{x+1}{3x+2}$</p>	0.50																					
	<p>4b b) $x = -1$</p>	0.50																					
<p>4c $\frac{x+1}{3x+2} = \frac{3}{7}; 2x=1; \text{then } x = \frac{1}{2}, \text{ rejected.}$</p>	0.75																						
IV	<p>1 Locate A, B and C For the line (d):</p> <table border="1"> <tr> <td>x</td> <td>5</td> <td>8</td> </tr> <tr> <td>y</td> <td>-3</td> <td>3</td> </tr> </table> 	x	5	8	y	-3	3	1.25															
x	5	8																					
y	-3	3																					

	2	$N(6,5; 0)$.	0.25
	3	ABC is right at B because...	1
	4	$M(3.5; 4)$.	0.50
	5	Coordinates of the vector \overrightarrow{MB} : $x_{\overrightarrow{MB}} = x_B - x_M = 1 - 3,5 = -2,5$; $y_{\overrightarrow{MB}} = y_B - y_M = -1 - 4 = -5$ Coordinates of the vector \overrightarrow{CN} : $x_{\overrightarrow{CN}} = x_N - x_C = 6,5 - 9 = -2,5$; $y_{\overrightarrow{CN}} = y_N - y_C = 0 - 5 = -5$. Then $\overrightarrow{MB} = \overrightarrow{CN}$ so N is the translate of C with respect to $T_{\overrightarrow{MB}}$.	1
	6	$\overrightarrow{MB} = \overrightarrow{CN}$, then $BMCN$ is a parallelogram and we have ABC is a right triangle, $[BM]$ is the median relative to the hypotenuse therefore $BM = \frac{AC}{2} = MC$, hence $BMCN$ is a rhombus.	1
V	1	(OM) is the perpendicular bisector of $[BC]$ because...	0.50
	2a	OMP and OIA are similar $\hat{O}IA = \hat{O}PM = 90^\circ$ and \hat{POI} is a common angle.	0.50
	2b	$\frac{OM}{OA} = \frac{MP}{AI} = \frac{OP}{OI}$; then $OA \times OP = OM \times OI$	0.50
	3a	The circle circumscribed about the triangle MCI , has a diameter $[MC]$ and $(MC) \perp (OC)$ then (OC) a tangent to (S')	0.50
	3b	The two triangles OIC and OCM are similar since : $\hat{OCM} = \hat{OIC} = 90^\circ$ and \hat{COI} is a common angle. $\frac{OI}{OC} = \frac{OC}{OM}$, then $OM \times OI = OC^2 = R^2$.	0.75
	3c	$OM \times OI = OA \times OP = R^2$, then $OP = \frac{R^2}{2R} = \frac{R}{2}$. O, A and P are fixed. The locus of M is the perpendicular at P to $[OA]$.	1
	4a	$BC = OM = R\sqrt{2}$. In the right triangle OPM : $MP^2 = OM^2 - OP^2 = 2R^2 - \frac{R^2}{4} = \frac{7R^2}{4}$; $MP = \frac{R\sqrt{7}}{2}$.	0.75
	4b	$\tan \hat{MAP} = \frac{MP}{AP} = \frac{\frac{R\sqrt{7}}{2}}{\frac{3R}{2}} = \frac{\sqrt{7}}{3}$.	0.25
	4c	$\hat{MAP} = \tan^{-1} \frac{\sqrt{7}}{3} = 41^\circ$ (rounded down) or 42° (rounded up).	0.25