

الاسم:
الرقم:مسابقة في مادة الرياضيات
المدة: ساعتان

عدد المسائل: ستة

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو لاختزان المعلومات أو لرسم البيانات. يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I) (1 1/2 pts)

Write each of the following numbers in the form of a fraction as simple as possible:

$$A = \frac{7}{3} - \frac{8}{3} \times \frac{5}{2} ; \quad B = \frac{\frac{5}{3} - 1}{1 - \frac{1}{6}} ; \quad C = \frac{8 \times 10^7 \times 1.5}{3 \times 10^9} .$$

II) (2 1/2 pts)

Given the two numbers X and Y:

$$X = \sqrt{32} - 3\sqrt{2} + 2\sqrt{18} ; \quad Y = \sqrt{50} - \sqrt{72} + 3\sqrt{2} .$$

1) Write X in the form $a\sqrt{2}$ and Y in the form $b\sqrt{2}$ where a and b are two integers to be calculated.

2) Deduce that $X \times Y = 28$.

3) Prove that the opposite table is a proportionality table.

X	$4\sqrt{3} + 2\sqrt{5}$
$4\sqrt{3} - 2\sqrt{5}$	Y

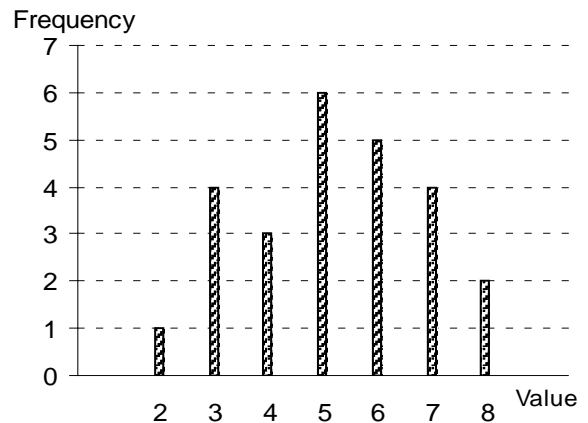
III) (2 1/2 pts)

The opposite bar graph represents a statistical series.

1) Calculate the total frequency.

2) Represent this series in a table showing the frequencies, and the relative frequencies in percentage.

3) Calculate the mean of this series.



IV) (2 ½ pts)

To buy two copybooks and one pen we must pay 2750 LL, and to buy four copybooks and three pens we must pay 7750 LL. The preceding information is translated into the

following system:
$$\begin{cases} 2x + y = 2750 \\ 4x + 3y = 7750 \end{cases}$$

- 1) What does x and y represent in this system?
- 2) Which information is translated by the equation $4x + 3y = 7750$?
- 3) Solve the preceding system, showing the followed steps in detail, to find the price of a copybook and the price of a pen.

V) (6 pts)

Consider in an orthonormal system of axes $x'Ox, y'Oy$, the points :

$$A(-2; 2); B(3; 1) \text{ and } E(0; -1).$$

- 1) Plot the points A, B and E.
- 2) Write an equation of the line (BE).
- 3) Knowing that $AB = \sqrt{26}$ and $BE = \sqrt{13}$, calculate AE and prove that the triangle ABE is an isosceles right triangle at E.
- 4) Let (C) be the circle circumscribed about triangle ABE. Calculate the radius of (C) and the coordinates of its center J.
- 5) Designate by F the image (translate) of A by the translation of vector \overrightarrow{EB} .
 - a) Prove that AEBF is a square.
 - b) Deduce that F is a point of (C).
 - c) Calculate the coordinates of F.

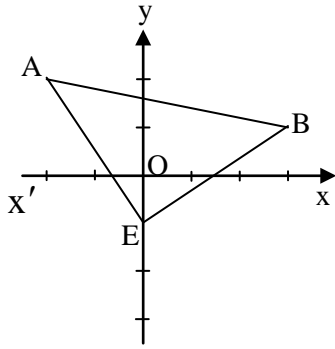
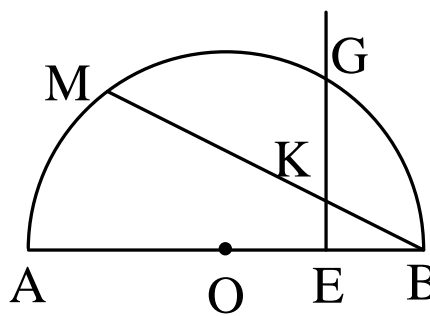
VI) (5 pts)

Consider a semi-circle (C) of diameter [AB], of center O and of radius R. Let E be the midpoint of segment [OB].

The perpendicular bisector of [OB] cuts (C) in G. Let K be a variable point on segment [EG]. The straight line (BK) cuts (C) in a second point M.

- 1) Draw a figure.
- 2) Prove that $OB = OG = GB$. Deduce the measure of the angle \widehat{BOG} .
- 3) Calculate, in terms of R, the area of triangle AGB.
- 4) a) Prove that the two triangles BEK and BMA are similar.
b) Deduce that $BK \times BM = BA \times BE$.
- 5) Designate by N the midpoint of [AM].
Prove that, when K describes [EG], N moves on a circle whose diameter is to be determined.

Questions	Eléments de réponses	Notes																											
I-	$A = \frac{7}{3} - \frac{20}{3} = \frac{-13}{3}$ $B = \frac{\frac{3}{6-1}}{6} = \frac{2}{3} \times \frac{6}{5} = \frac{4}{5}$ $C = \frac{4}{100} = \frac{2}{50} = \frac{1}{25}$	<p>1/2</p> <p>1/2</p> <p>1/2</p>																											
II-	<p>1- $X = \sqrt{32} - 3\sqrt{2} + 2\sqrt{18} = 4\sqrt{3} - 3\sqrt{2} + 6\sqrt{2} = 7\sqrt{2}$.</p> <p>$Y = 5\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = 2\sqrt{2}$.</p> <p>2- $x \times y = 28$.</p> <p>3- $(4\sqrt{3} - 2\sqrt{5})(4\sqrt{3} + 2\sqrt{5}) = 28 = xy$.</p>	<p>3/4</p> <p>1/2</p> <p>1/4</p> <p>1</p>																											
III-	<p>1- $N = 25$.</p> <p>2-</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th>valeurs</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> <th>7</th> <th>8</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>effectifs</td> <td>1</td> <td>4</td> <td>3</td> <td>6</td> <td>5</td> <td>4</td> <td>2</td> <td>25</td> </tr> <tr> <td>fréquences en %</td> <td>4</td> <td>16</td> <td>12</td> <td>24</td> <td>20</td> <td>16</td> <td>8</td> <td>100</td> </tr> </tbody> </table> <p>3- $\bar{X} = \frac{2 \times 1 + 3 \times 4 + 4 \times 3 + 5 \times 6 + 6 \times 5 + 7 \times 4 + 8 \times 2}{25} = \frac{130}{25} = 5.2$</p>	valeurs	2	3	4	5	6	7	8	Total	effectifs	1	4	3	6	5	4	2	25	fréquences en %	4	16	12	24	20	16	8	100	<p>1/2</p> <p>1/4</p> <p>3/4</p> <p>1</p>
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IV-	<p>1- x est le prix d'un cahier ; y est le prix d'un stylo.</p> <p>2- Le prix de 4 cahiers et de 3 stylos est 7750 LL.</p> <p>3- $-4x - 2y = -5500$</p> $\frac{4x + 3y = 7750}{y = 2250 ; x = 250}$ <p>Le prix d'un cahier est 250 LL et celui d'un stylo est 2250 LL.</p>	<p>1/2</p> <p>1/2</p> <p>1 1/2</p>																											

Questions	Eléments de réponses		Notes
V-	1-	<div style="text-align: center;">  </div> <p>2- Equation de (BE) :</p> $a = \frac{2}{3} ; b = -1 \text{ d'où } y = \frac{2}{3}x - 1.$ <p>3- $AE = \sqrt{13}$; ABE est un triangle rectangle isocèle.</p> <p>4- Rayon de (C) = $\frac{\sqrt{26}}{2}$; $J\left(\frac{1}{2}; \frac{3}{2}\right)$</p> <p>5-a) AEBF est un carré . b) F est un point de (C). c) F(1 ; 4).</p>	<p>$\frac{3}{4}$</p> <p>1</p> <p>1$\frac{1}{4}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
VI-	1-	<p>Figure</p> <p>2- $OB = OG = BG = R$ $\widehat{BOG} = 60^\circ$</p> <p>3- L'aire de AGB = $\frac{R^2\sqrt{3}}{2}$.</p> <div style="text-align: center;">  </div> <p>4-a) BEK et BMA sont semblables car ... b) $\frac{BE}{BM} = \frac{EK}{MA} = \frac{BK}{BA}$. alors $BK \times BM = BA \times BE$</p> <p>5- N se déplace sur le cercle de diamètre [AO].</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>1</p>