

عدد المسائل : ستة	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:
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ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.  
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

**I- (3 points)**

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question ,and give with justification, its corresponding answer .

N <sup>0</sup>	Question	Proposed answers		
		a	b	c
1	In an orthonormal system, the two lines with equations $y = 2x + 3$ and $2y + x = 1$ are...	intersecting at the point (1,5)	perpendicular	parallel
2	The original price of an article is 30 000 LL ,and its price after the discount is 27 600 LL . The percentage of discount is...	10 %	18 %	8 %
3	A triangle ABC is right at B. If $AB=3$ and $\angle ACB = 40^\circ$ , then $AC = \dots$	$\frac{3}{\sin 40^\circ}$	$3 \sin 40^\circ$	$\frac{\sin 40^\circ}{3}$
4	The natural numbers, solutions of $3x-1 < 8$ are...	1 ; 2 ; 3	0 ; 1 ; 2 ; 3	0 ; 1 ; 2
5	A triangle ABC is right at A and M is the midpoint of [BC]. If $AM = AB = 6$ then $AC = \dots$	12	$6\sqrt{3}$	$9\sqrt{3}$
6	ABCD is a parallelogram and E is the translate of D under the translation with vector $\overrightarrow{BA}$ . Therefore...	C is the midpoint of [DE]	$\overrightarrow{DB} + \overrightarrow{DE} = \overrightarrow{DA}$	$\overrightarrow{DE} + \overrightarrow{DA} = \overrightarrow{DB}$

**II-(2 points)**

Consider the three numbers A , B and C :

$$A = (\sqrt{5} + 1)^2 + (\sqrt{5} - 1)^2 ; B = \frac{3}{4} + \frac{5}{4} \times \frac{7}{15} \text{ and } C = \frac{(10\sqrt{3})^4}{25 \times 10^3 \times 3 + 10^3 \times 6 \times 37.5} .$$

In the following questions , the steps of calculation must be shown

- 1) Prove that A is a natural number.
- 2) Write B as a fraction in its simplest form.
- 3) Prove that C is a decimal number.

**III- (2 points)**

In what follows, are the scores of a student in five tests: 10 ; 8 ; 13 ; x and y.

The difference between x and y is 7. The average ( mean) of these five scores is 12.

- 1) Write a system of two equations with two unknowns modeling the given situation .
- 2) Solve the obtained system .

**IV- (3 points)**

Consider the expressions:  $A(x) = 4x^2 - 9$  and  $B(x) = (2x - 3)^2 - (2x - 3)(x - 5)$ .

- 1) Factorize  $A(x)$ .
- 2) a- Verify that  $B(x) = (2x - 3)(x + 2)$ .  
b- Solve the equation  $B(x) = 0$ .
- 3) Consider the expression  $F(x) = \frac{A(x)}{B(x)}$ .
  - a. For what values of  $x$ ,  $F(x)$  is defined ?
  - b. Simplify  $F(x)$  and solve  $F(x) = 3$ .
  - c. Calculate  $F(\sqrt{5})$  and write the answer in the form  $a - b\sqrt{5}$ . ( $a$  and  $b$  are natural numbers)

**V- (5 points)**

In an orthonormal system of axes  $x'Ox$ ,  $y'Oy$ , where the unit of length is the centimeter, consider the points  $A(1;-2)$ ,  $B(2;1)$ ,  $C(5;0)$  and the line  $(d)$  with equation  $y = 3x - 5$ .

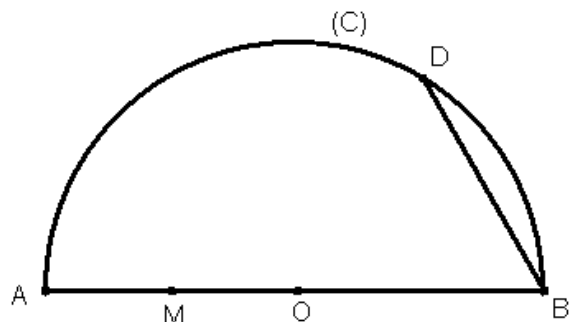
- 1) a. Verify that  $(d)$  is passing through  $A$  and  $B$ .  
b. Plot the points  $A$ ,  $B$ ,  $C$  and draw  $(d)$ .
- 2) a. Determine the equation of the line  $(BC)$ .  
b. Calculate the lengths  $AB$  and  $BC$ .  
c. Prove that  $ABC$  is a right isosceles triangle with vertex  $B$ .
- 3) Let  $(G)$  be the circle circumscribed about the triangle  $ABC$  and  $D$  is the point defined by  $\overline{BD} = \overline{AC}$ .
  - a. Calculate the coordinates of  $D$ .
  - b. Prove that  $(DB)$  is tangent to  $(G)$ .
- 4)  $(d')$  is the translate of  $(d)$  under the translation with vector  $\overline{AC}$ . Find the equation of  $(d')$ .

**VI- (5 points)**

Consider a semicircle  $(C)$  with centre  $O$  and radius  $R$ .  $[AB]$  is the diameter of  $(C)$  and  $D$  is a point on  $(C)$  so that  $BD = R$ .

Let  $M$  be the midpoint of  $[OA]$ . The perpendicular bisector of  $[OA]$  intersects  $[AD]$  at  $E$ ,  $(BD)$  at  $F$  and  $(C)$  at  $K$ .

- 1) Reproduce and complete the figure.
- 2) Calculate the angles of the triangle  $ABD$ , then calculate  $AD$  in terms of  $R$ .
- 3) a. Prove that the two triangles  $ADB$  and  $FMB$  are similar.  
b. Calculate  $BF$  in terms of  $R$ .
- 4)  $(BE)$  intersects  $(AF)$  at  $J$ . Show that  $J$  is on  $(C)$ .
- 5) Prove that the points  $B$ ,  $M$ ,  $J$  and  $F$  are on the same circle whose center and radius should be determined.
- 6) a. Show that the triangle  $OKA$  is equilateral.  
b. Prove that  $(OK)$  passes through the midpoint of  $[AF]$ .



الاسم:  
الرقم:مسابقة في مادة الرياضيات  
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مشروع معيار التصحيح

	Part of the Q	Answer	Mark
<b>I</b> (3pts)	1	$(2)(-0.5) = -1$ , therefore (b)	<b>0.5</b>
	2	$30\,000 - 27\,600 = 2400 = \frac{30\,000 \times x}{100}$ ; $x=8$ (8 %), therefore (c)	<b>0.5</b>
	3	$\frac{3}{\sin 40^\circ}$ therefore (a)	<b>0.5</b>
	4	0;1;2 therefore (c)	<b>0.5</b>
	5	ABM is equilateral, then $BC=12$ and $AC = \frac{12\sqrt{3}}{2} = 6\sqrt{3}$ . therefore (c)	<b>0.5</b>
	6	DEAB is a parallelogram, $\overline{DB} + \overline{DE} = \overline{DA}$ therefore (c)	<b>0.5</b>
	Part of the Q	Answer	Mark
<b>II</b> (2pts)	1	$A=12$	<b>0.75</b>
	2	$B = \frac{4}{3}$	<b>0.5</b>
	3	$C = 0.3$	<b>0.75</b>
	Part of the Q	Answer	Mark
<b>III</b> (2pts)	1	$\frac{x+10+8+13+y}{5} = 12$ ; so, $x + y + 31 = 60$ Thus, $x + y = 29$ The system is $\begin{cases} y = x + 7 \\ x + y = 29 \end{cases}$	<b>1.25</b>
	2	$x = 11$ , so $y = 18$ .	<b>0.75</b>

	Part of the Q	Answer	Mark
<b>IV</b> (3pts)	<b>1</b>	$A(x) = (2x - 3)(2x + 3)$	<b>0.5</b>
	<b>2.a</b>	$B(x) = (2x - 3)(x + 2)$ .	<b>0.5</b>
	<b>2.b</b>	$B(x) = 0$ $x = -2$ or $x = \frac{3}{2}$	<b>0.5</b>
	<b>3. a</b>	$F(x)$ is defined $x \neq -2$ and $x \neq \frac{3}{2}$	<b>0.25</b>
	<b>3.b</b>	$F(x) = \frac{2x+3}{x+2}$ $F(x) = 3$ ; $x = -3$ .	<b>0.75</b>
	<b>3.c</b>	$F(\sqrt{5}) = \frac{2\sqrt{5}+3}{2+\sqrt{5}} = 4 - \sqrt{5}$ .	<b>0.5</b>

<b>V</b> (5pts)	1. a	A and B are two points of (d)	<b>0.5</b>
	1. b		<b>0.75</b>
	2. a	$(BC) : y = -\frac{1}{3}x + \frac{5}{3}$	<b>0.75</b>
	2. b	$AB = \sqrt{10} ; BC = \sqrt{10}$	<b>0.75</b>
	2. c	$AB = \sqrt{10} ; BC = \sqrt{10}$ and $(AB)$ is perpendicular to $(BC)$ then $ABC$ is a right isosceles triangle at $B$ .	<b>0.5</b>
	3.a	$\overline{AC}(4;2); \overline{BD}(x; y); \overline{BD}(x-2; y-1)$ then $D(6;3)$	<b>0.5</b>
	3.b	$(BD)$ is perpendicular to $(IB)$	<b>0.75</b>
	4	$(d') : y = 3x - 15$	<b>0.5</b>

<b>VI</b> (5pts)	1		<b>0.5</b>
	2	$\sphericalangle ADB = 90^\circ, \sphericalangle ABD = 60^\circ, \sphericalangle BAD = 30^\circ : AD = R\sqrt{3}$ .	<b>1</b>
	3.a	The two triangles are right, and angle $\sphericalangle B$ is common	<b>0.5</b>
	3.b	$BF=3R$	<b>0.5</b>
	4	$(BE)$ is the third altitude in the triangle $AFB$ , then $ABJ$ is a right triangle at $J$ , then $J$ belongs to the circle	<b>0.5</b>
	5	$\sphericalangle FJB = \sphericalangle FMB = 90^\circ$ $B, M, J$ and $F$ are on the same circle. with center midpoint of $[BF]$ , radius = $\frac{3R}{2}$	<b>0.75</b>
	6.a	$\triangle AKO$ is an equilateral triangle since $KA=KO=OA=R$ .	<b>0.5</b>
	6.b	$(OK)$ parallel to $(BF)$ , $O$ midpoint of $[AB]$ , $(OK)$ passes through the midpoint of $[AF]$ .	<b>0.75</b>